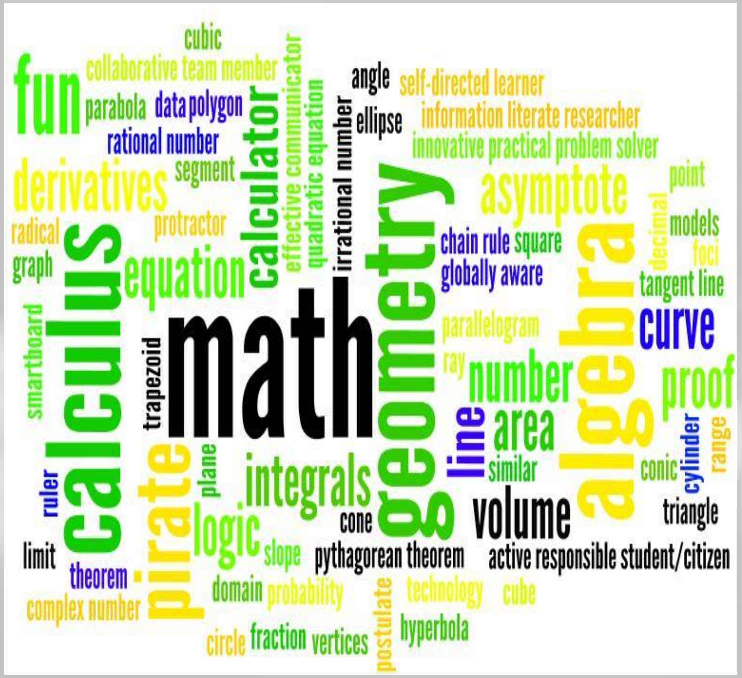
How to Break the Habit of Teaching Only Procedural Mathematics



Facilitated by:
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Objectives

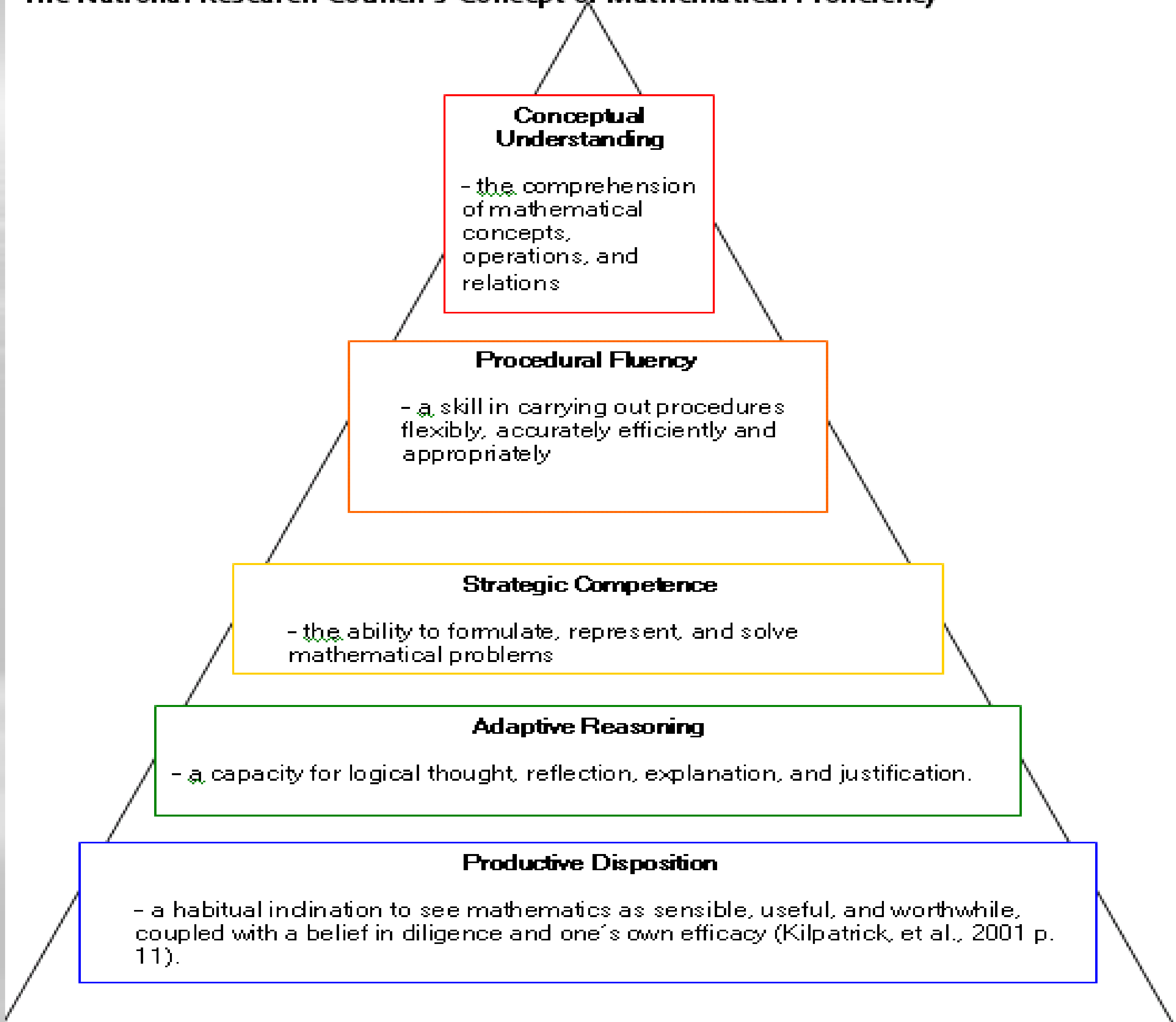
- Understand the drawbacks of teaching only procedural mathematics
- Analyze how we learned mathematics
- Practice identifying key concepts behind common procedures
- Develop techniques for creating lesson/unit plans that are rooted in conceptual understanding and progress into procedures

VS.

Having only a procedural understanding of mathematics is like a cook who has to carefully follow a recipe.

Having a conceptual understanding of mathematics is like the chef who can take ingredients and create a dish!

The National Research Council's Concept of Mathematical Proficiency



Conceptual Understanding

The comprehension of mathematical concepts operations and relations

Comprehending mathematical concepts, operations, and relations – knowing what mathematical symbols, diagrams, and procedures mean. Enables you to connect ideas to what you already know. Supports retention and prevents common errors.

Source: http://mathequality.wordpress.com/2012/06/25/nrcs-five-strands-of-mathematical-proficiency/

Procedural Fluency

The skill of carrying out procedures flexibly, accurately, efficiently and appropriately

> Carrying out mathematical procedures mentally, or with paper and pencil, and knowing when and how to use these procedures flexibly, accurately, efficiently, and appropriately. Learning procedures can strengthen and develop mathematical understanding, while understanding makes it easier to learn skills; these complement each other.

Source: http://mathequality.wordpress.com/2012/06/25/nrcs-five-strands-of-mathematical-proficiency/

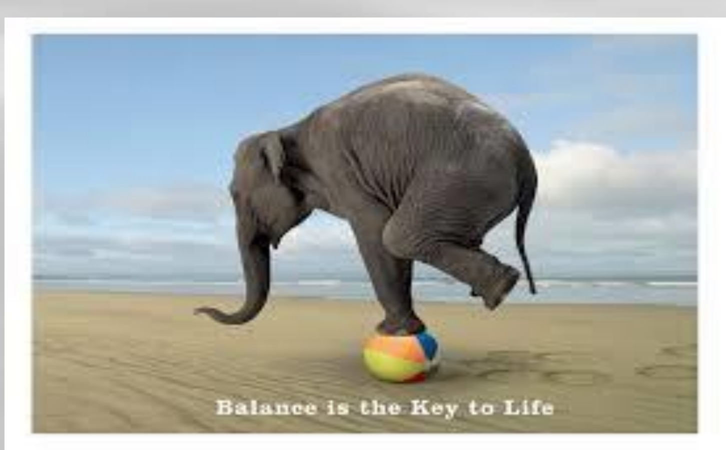
Why One without the Other?

Why does most mathematics instruction focus on procedural fluency without applying a balanced approach for developing conceptual understanding?

Balancing Act

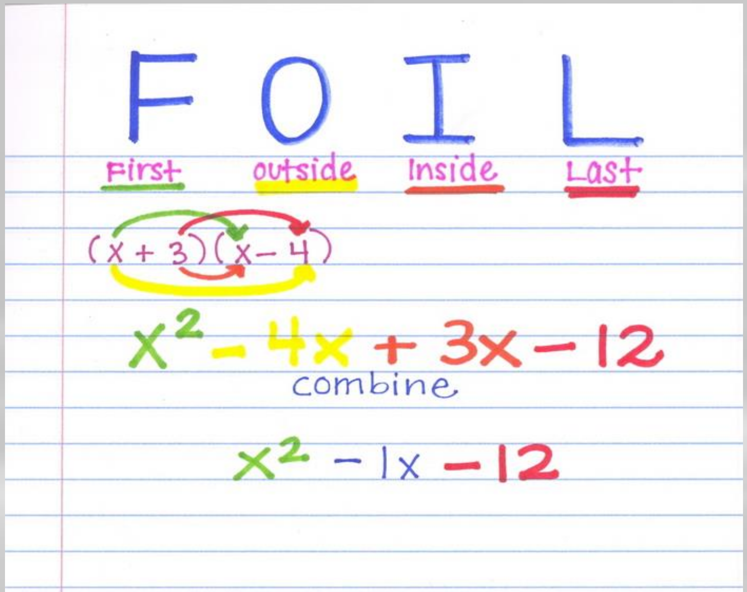
There needs to be a balance between instruction for conceptual understanding and procedural fluency. Studies have shown that a balance between these two leads to long-term retention of both…particularly if connections between them are made explicit[ly]"

Source: A.H. Schoenfeld (1988) When good teaching leads to bad results: The disasters of well taught mathematics classes



FOIL

dreamstime.com



F O I L
First outside Inside Last
(x + 3)(x – 4)
x² – 4x + 3x – 12
combine
x² – 1x – 12

Division by Concept

Divide 15,832 by 9

- You could do this with the four basic steps, which could be mastered or could leave the door open for numerous mistakes

OR

- You could figure out how many 9's you can take out of the number.

Instructional Shifts

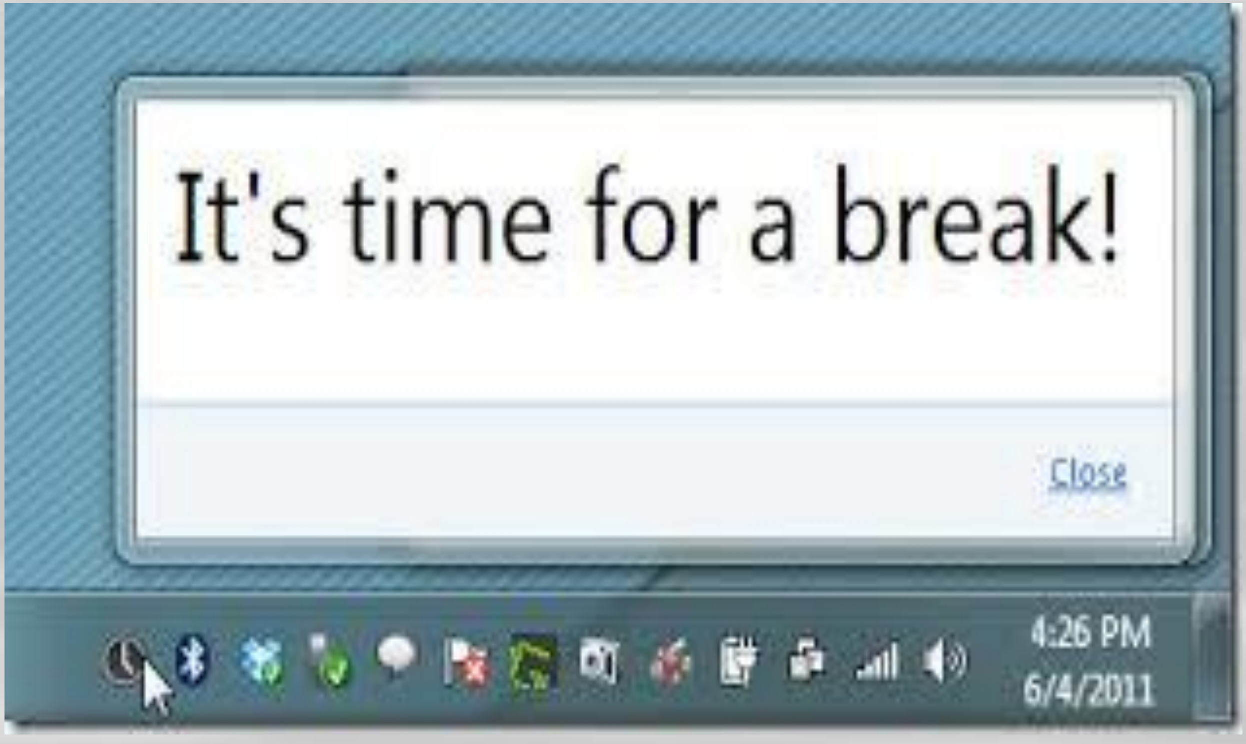
Deep Understanding

- "Students who memorize facts or procedures without understanding often are not sure when or how to use what they know and such learning is often quite fragile." (NCTM's Principles and Standards for School Mathematics 2000)

- Is needed because understanding ***can*** lead to memorization, memorization ***does not*** lead to understanding

Fluency

- Fluency refers to having efficient and accurate methods for computing. Students exhibit…fluency when they demonstrate *flexibility* in the computational methods they choose, *understand* and can explain these methods, and produce accurate answers *efficiently*.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How to Identify the Concepts

- Begin with any mathematical procedure and then ask yourself **“Why?”**
- The answer to the why could be a definition, an understanding of a connection to another mathematical principle or property, or could be determined with logical reasoning.

Now You Try It!

- Can you identify the concept behind the procedure of dividing fractions. Some instructors teach it as “Keep, Change, Flip?”

Example: $\frac{1}{4} \div \frac{2}{3} = \;\Rightarrow\; \frac{1}{4} \times \frac{3}{2} =$

Instruction for Conceptual Understanding

- Determine the math skill or even procedure the your lesson will be about
- Identify what the underlying concept
- Develop a task, activity, or whole class discussion
- Allow students to think, conjecture and discover procedures
- Finally allow students adequate time to practice their procedures

4 Beliefs for Math Teachers

1. Math…is a web of interrelated concepts and procedures
2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures
4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them

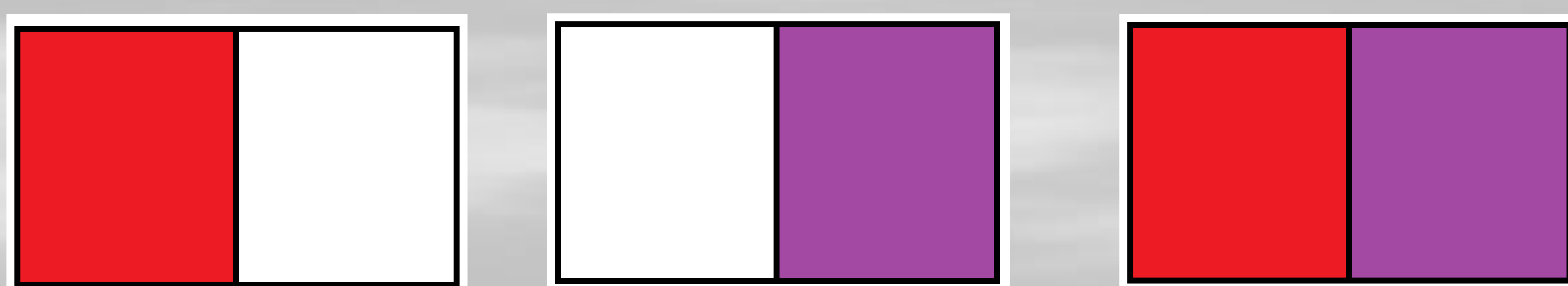
Source: Assessing prospective elementary school teacher's beliefs about mathematics and mathematics learning
Ambrose, Clement, Philipp, & Chauvot (2004)

Promoting Deep Understanding

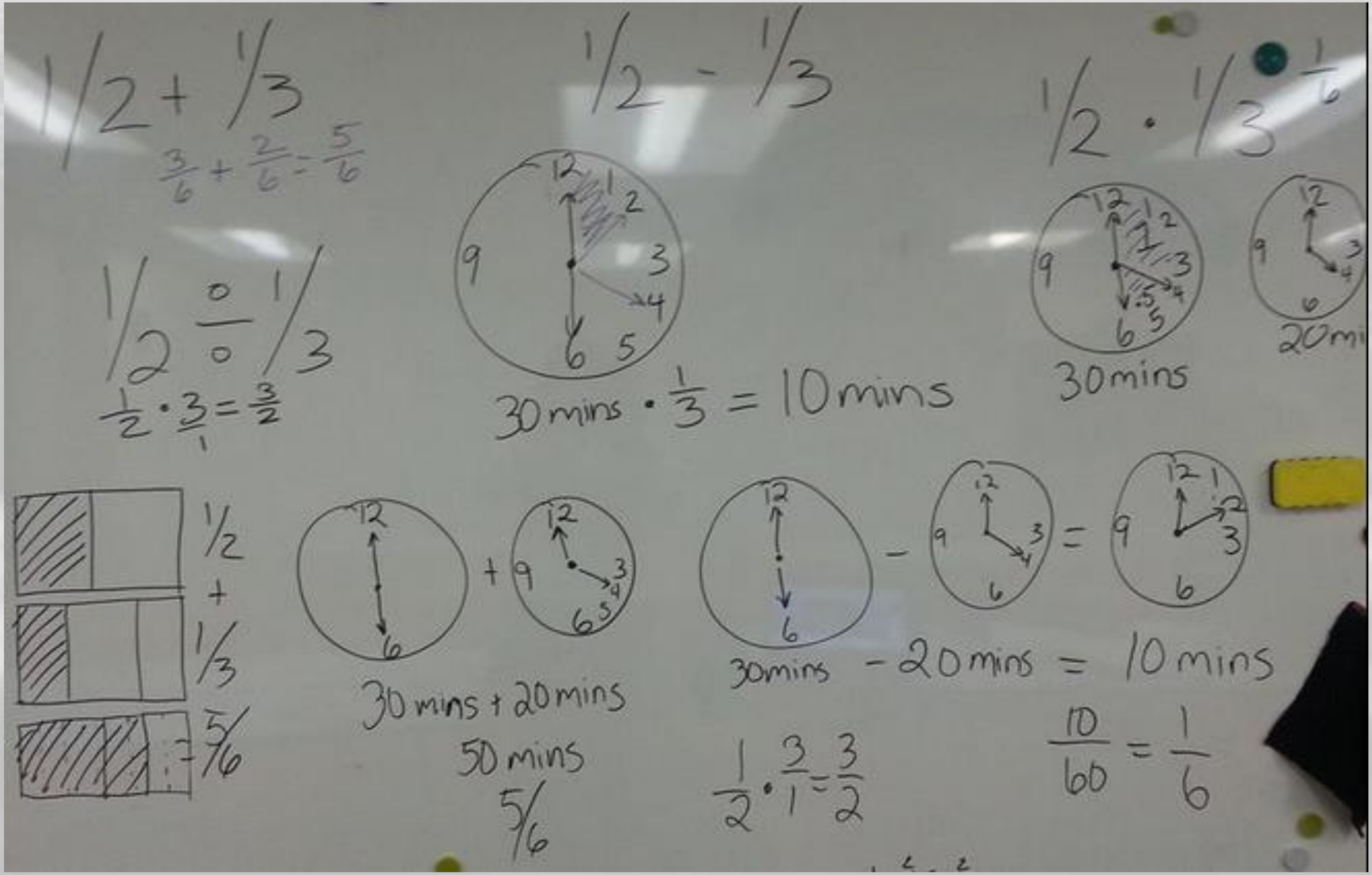
1. Pictorial models
2. Conceptual questioning
3. Writing/speaking about understanding
4. Identifying misconceptions and correcting them
5. Worked examples
6. Create activities where students test theories
7. Create counter examples to help fix misconceptions
8. Focus on meaning

Example

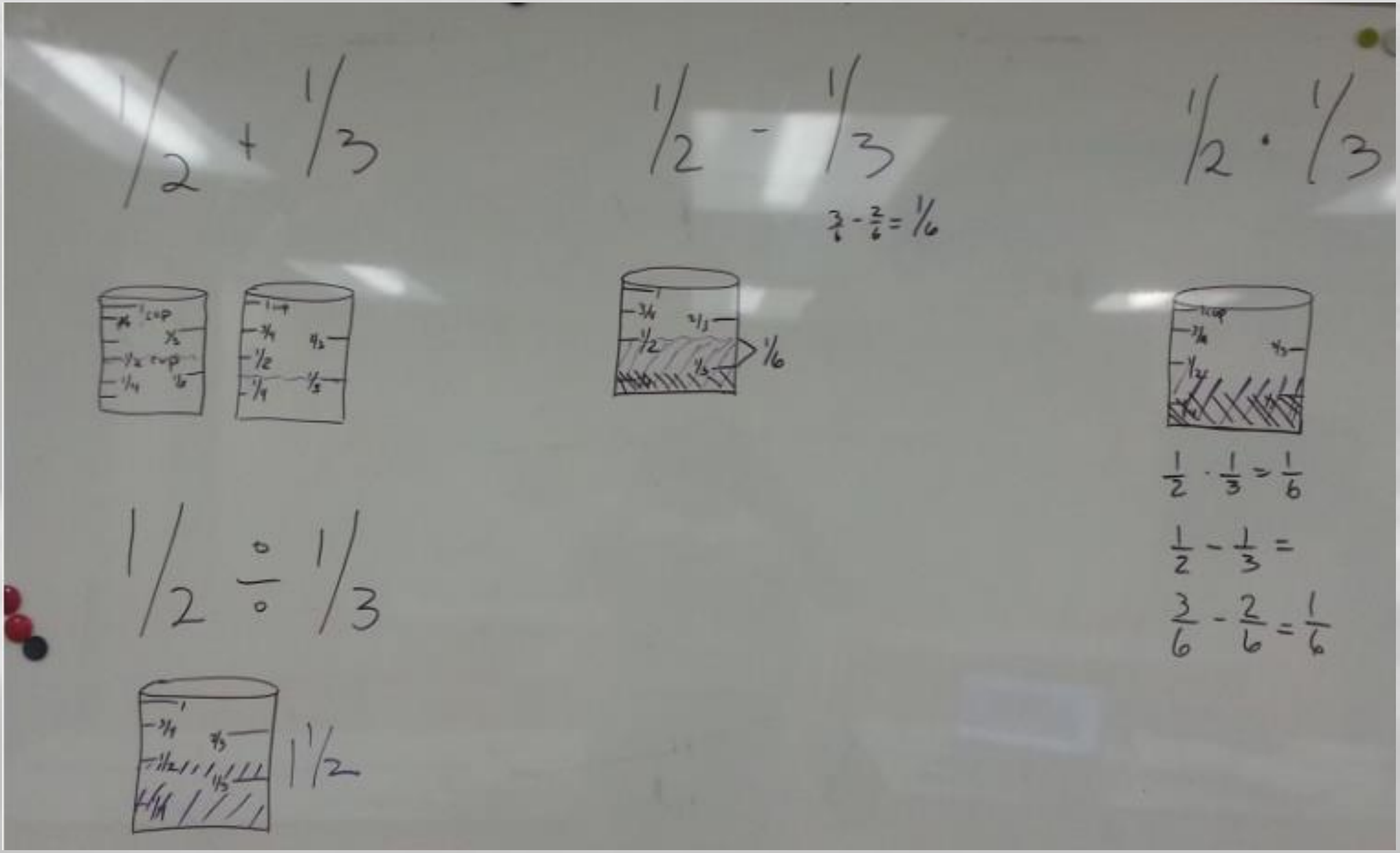
$$\frac{1}{2} + \frac{1}{2} =$$



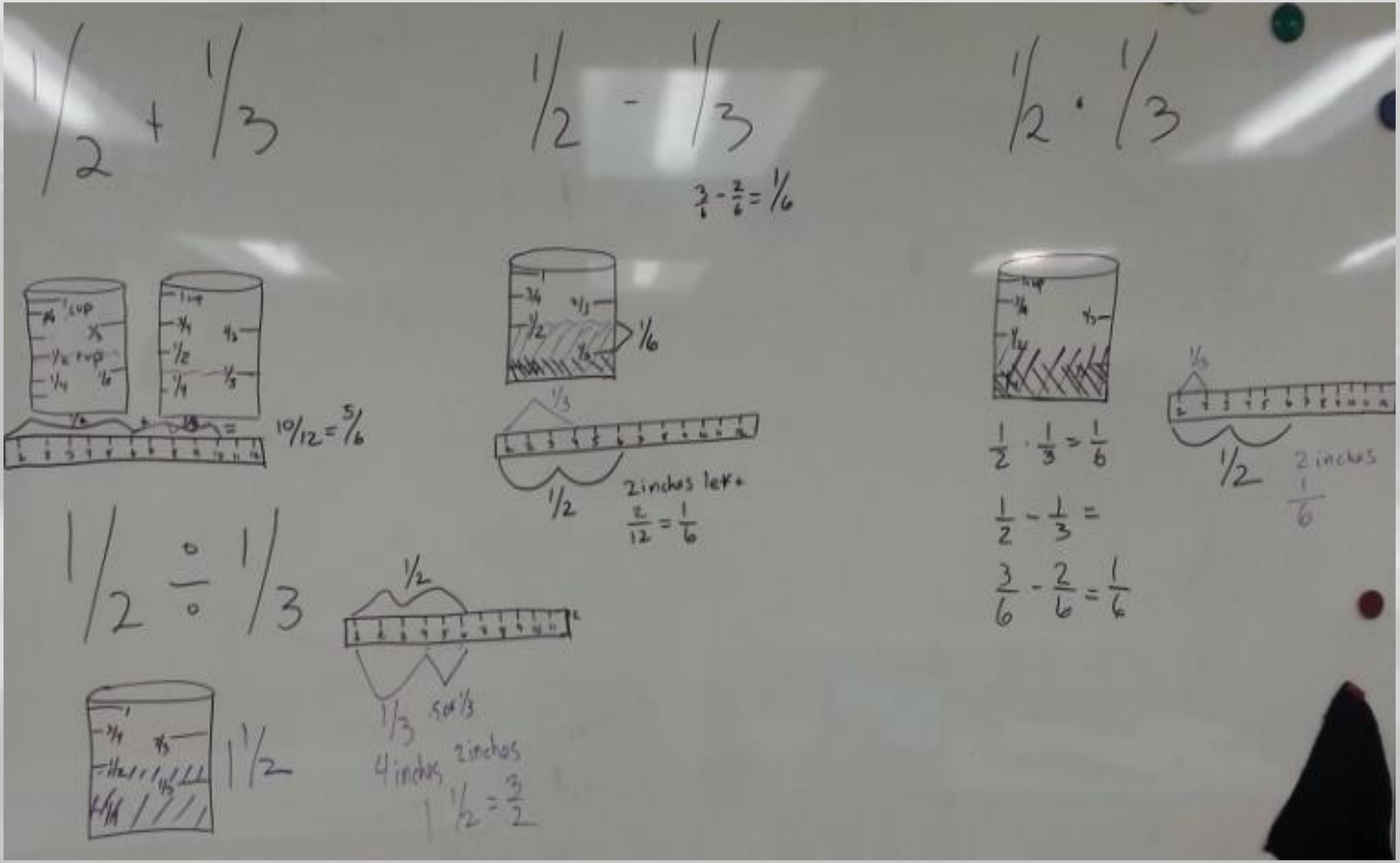
You could have….



Or…



Or a ruler…



1/2 + 1/3
10/12 = 5/6
1/2 - 1/3
3/6 - 2/6 = 1/6
2 inches left
2/12 = 1/6
1/2 · 1/3
1/2 · 1/3 = 1/6
3/6 - 2/6 = 1/6
2 inches
1/2 ÷ 1/3
1 1/2
4 inches
2 inches
1 1/2 = 3/2

Reminder

- Remember that you are trying to provide students the opportunity to create a dish not lock them down to a particular recipe.

- Stop using the excuse, “We don’t have enough time to teach ‘why?’”

Contact Information

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